

$$y = f(x)$$

$\nwarrow$ Dependent variable $\uparrow$ Independent variable

Diff. eqⁿ: An eqⁿ. involving derivatives of one or more dependent var^s. w.r.t. one or more independent variables.

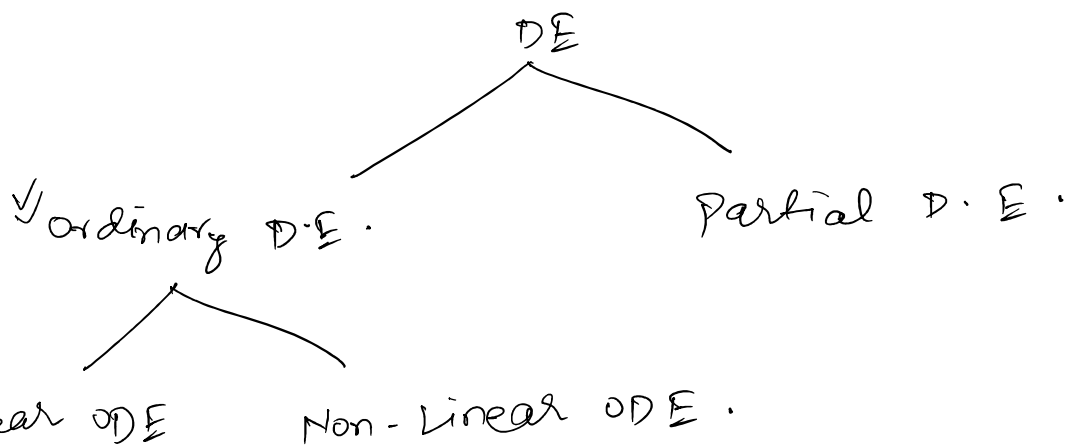
$$f\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \dots, \frac{d^ny}{dx^n}\right) = 0$$

expresses relation b/w dependent & independent variable & their derivative of any order.

Eg: $\frac{dy}{dx} = x + 3$

ODE

O → 1
D → 1



Linear ODE: A linear ODE of order n in D.V. y & I.V. x , is an eqⁿ. that can be expressed as

$$a_0(x) \frac{d^ny}{dx^n} + a_1(x) \frac{d^{n-1}y}{dx^{n-1}} + \dots + a_{n-1}(x) \frac{dy}{dx} + a_n(x)y = b(x)$$

or

If D.V. y & its derivative occur in first degree ONLY AND no products of y & its derivative are present.

Eg. 1) $\frac{d^2y}{dx^2} + 5 \frac{dy}{dx} + 6y = 0$

LODE. $0 \rightarrow 2$
 $1 \rightarrow 1$

2) $\frac{d^2y}{dx^2} + 5 \frac{dy}{dx} + 6y^3 = 0$

HL ODE. $0 \rightarrow 2$
 $1 \rightarrow 1$

3) $y \frac{d^2y}{dx^2} + 5 \frac{dy}{dx} + 6y = 0$

HL ODE. $0 \rightarrow 2$
 $1 \rightarrow 1$

4) $x^2 \frac{d^2y}{dx^2} + 5 \frac{dy}{dx} + 6y = 0$

$\frac{d^2y}{dx^2} \rightarrow y''$
 $y^{(2)}$

5) $yy' + 2y = 0$ NLODE

6) $y'' + 5(y')^2 + 6y = 0$ HLODE

Order of DE: highest ordered derivative in a D.E.

Degree of DE: Power of "that" highest order derivative.

Pick the highest differential: $\left(\frac{d^\theta y}{dx^\theta} \right)^{\text{degree}}$ $\theta \rightarrow \text{order}$

Eg. 1) $\left(\frac{d^2y}{dx^2} \right)^3 + \left(\frac{dy}{dx} \right)^4 + y^6 + x = 0$

H. D. $\left(\frac{d^2y}{dx^2} \right)^3 \rightarrow \text{degree}$

$0 \rightarrow 2$

$1 \rightarrow 3$

2) $\frac{d^2y}{dx^2} + xy \left(\frac{dy}{dx} \right)^3 = 0$

NL

$0 \rightarrow 2$

$1 \rightarrow 1$

3) $\frac{d^4y}{dx^4} + 5 \left(\frac{d^2y}{dx^2} \right)^2 + 3 \frac{dy}{dx} - y = 8$

NL

$0 \rightarrow 4$

$$4) \frac{dy}{dx} = \sin(x+y) \quad \text{NL} \quad \begin{matrix} O \rightarrow 1 \\ D \rightarrow 1 \end{matrix} \quad D \rightarrow 1$$

$$5) \sin\left(\frac{dy}{dx}\right) = x+y \quad \text{NL} \quad \begin{matrix} O \rightarrow 1 \\ D \rightarrow \text{not defined} \end{matrix}$$

$$6) \frac{dy}{dx} + \frac{xy}{\frac{dy}{dx}} = 0$$

$$\left(\frac{dy}{dx}\right)^2 + xy = 0 \quad \text{NL} \quad \begin{matrix} O \rightarrow 1 \\ D \rightarrow 2 \end{matrix}$$

NOTE: To find order & degree, make the DE free from fraction & radicals of D.V. y.

$$7) \frac{d^2y}{dx^2} = \sqrt{\frac{dy}{dx} + 1}$$

$$\left(\frac{d^2y}{dx^2}\right)^2 = \frac{dy}{dx} + 1$$

$$\begin{matrix} \text{NL} \\ O \rightarrow 2 \\ D \rightarrow 2 \end{matrix}$$

$$8) \frac{dy}{dx} = \sqrt{y}$$

$$\left(\frac{dy}{dx}\right)^2 = y$$

$$\begin{matrix} O \rightarrow 1 \\ D \rightarrow 2 \end{matrix}$$

$$9) \frac{dy}{dx} = \sqrt{x}$$

$$\begin{matrix} O \rightarrow 1 \\ D \rightarrow 1 \end{matrix}$$

$$10) \frac{d^5y}{dx^5} = y^{2/3}$$

$$\Rightarrow \left(\frac{d^5y}{dx^5}\right)^3 = y^2$$

$$\begin{matrix} O \rightarrow 5 \\ D \rightarrow 3 \end{matrix}$$

$$y^2 = x + c$$